Applications of Representation Theory

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What is Representation Theory?

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A **group** is any collection of transformations closed under composition and inverses.

Representation Theory gets its name from the tradition of representing groups as set of matrices over a vector space.

Symmetry Groups

Groups are usually used to measure symmetries.

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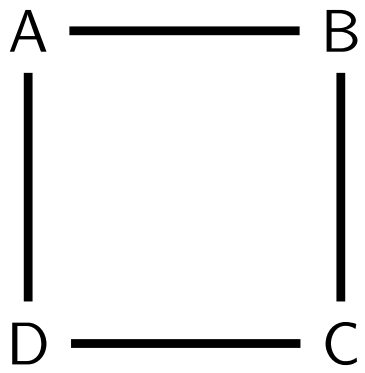
For example, consider the symmetries of the square D_4:



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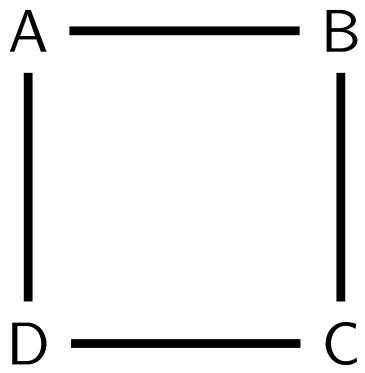

Id

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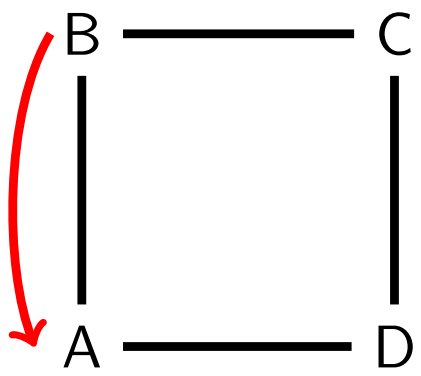

r_1

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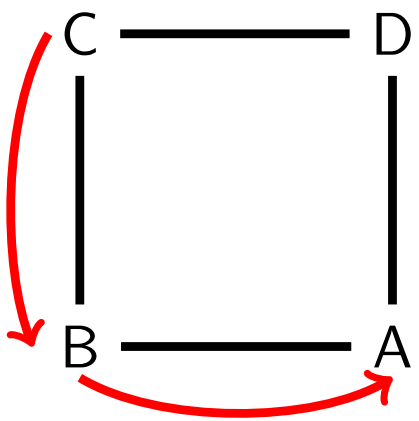

r_2

Symmetry Groups

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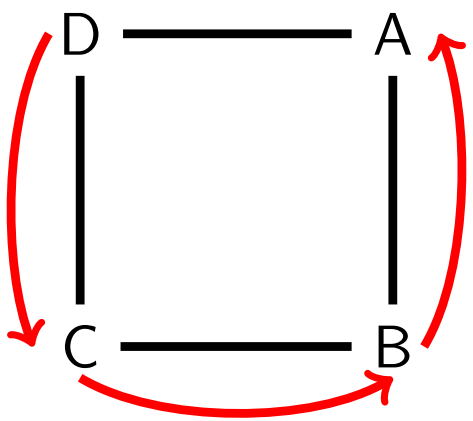

r_3

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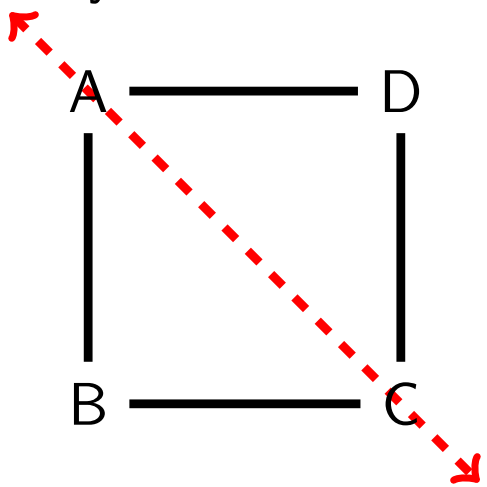

s_1

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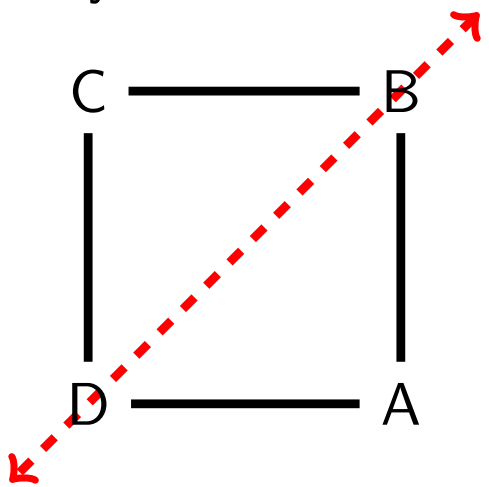

s_2

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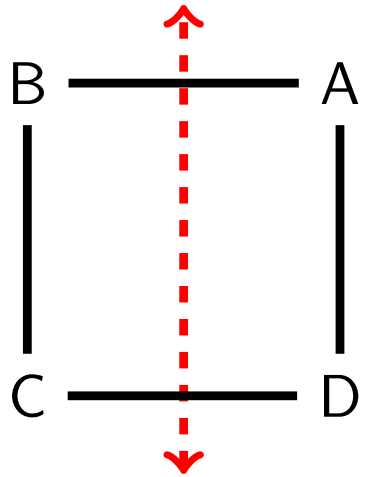

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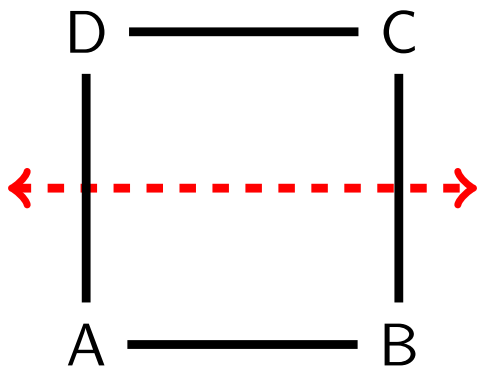

S_4

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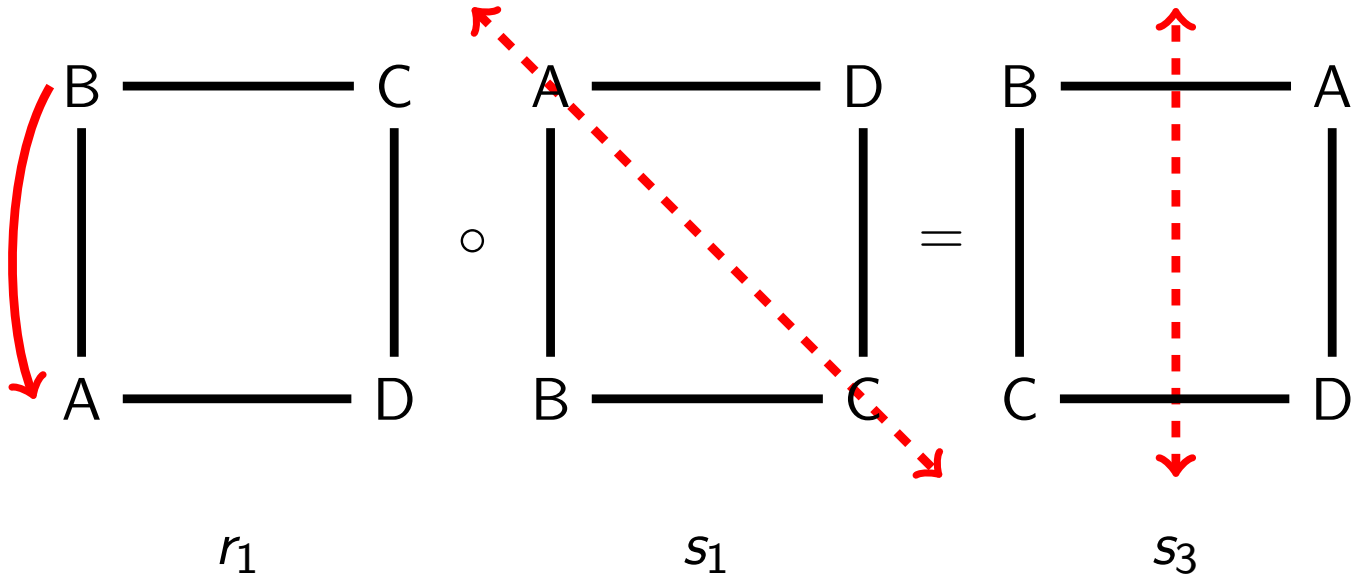

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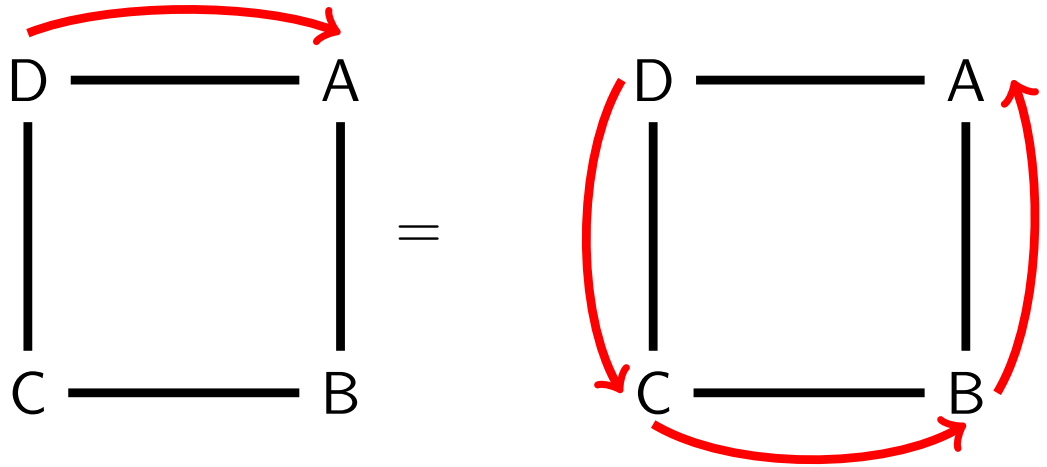

r_1^{-1} r_3

2014 Π Summer Graduate Program

Modern Applications of Representation Theory
University of Chicago

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Modern Applications of Representation Theory
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- Representation Theory in Cyro-Electron Microscopy
- Representation Theory in Computational Complexity
- Representation Theory in Digital Signal Processing
- Representation Theory in Fast Matrix Multiplication
- Representation Theory in Machine Learning (and Pattern Recognition)
- Representation Theory in Quantum Computation (and Holographic Algorithms)
- Representation Theory and Random Processes
- Representation Theory in Compressive Sensing
- Representation Theory in Identity Management
- Representation Theory in Phylogenetics
- Representation Theory in Statistics

Other Applications of Representation Theory

Other Applications of Representation Theory

- Telephone network designs
- Robotics
- Radar/Antenna Design
- Stereo Systems
- Error-Correcting Codes
- Computer Security
- Crystallography
- **Voting**

Voting

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- Candidate 3 : A Republican

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An exit poll survey was conducted, and voters were asked to rank the three candidates in order of preference.

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An exit poll survey was conducted, and voters were asked to rank the three candidates in order of preference. The findings were:

$$\mathbf{p} = 3\begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array} + 2\begin{array}{|c|}\hline 1\\\hline 3\\\hline 2\\\hline\end{array} + 0\begin{array}{|c|}\hline 2\\\hline 1\\\hline 3\\\hline\end{array} + 2\begin{array}{|c|}\hline 2\\\hline 3\\\hline 1\\\hline\end{array} + 0\begin{array}{|c|}\hline 3\\\hline 1\\\hline 2\\\hline\end{array} + 4\begin{array}{|c|}\hline 3\\\hline 2\\\hline 1\\\hline\end{array}$$

Voting

Who is the winner of the election?

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Candidate	First Choice	Second Choice	Last Choice
1	5	0	6
2	2	7	2
3	4	4	3

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Surprisingly, it depends more on the voting method, than the voters' profile.

Voting Paradox

Candidate	First Choice	Second Choice	Last Choice
1	5	0	6
2	2	7	2
3	4	4	3

Candidate	"Vote for Your Favorite"
1	
2	
3	

Voting Paradox

Candidate	First Choice	Second Choice	Last Choice
1	5	0	6
2	2	7	2
3	4	4	3

Candidate	"Vote for Your Favorite"
1	5
2	2
3	4

Voting Paradox

Candidate	First Choice	Second Choice	Last Choice
1	5	0	6
2	2	7	2
3	4	4	3

Candidate	"Vote for Your Least Favorite"
1	
2	
3	

Voting Paradox

Candidate	First Choice	Second Choice	Last Choice
1	5	0	6
2	2	7	2
3	4	4	3

Candidate	"Can I Have Two Votes?"
1	
2	
3	

Voting Paradox

Candidate	First Choice	Second Choice	Last Choice
1	5	0	6
2	2	7	2
3	4	4	3

Candidate	"Can I Have Two Votes?"
1	5
2	9
3	8

Voting Paradox

Candidate	First Choice	Second Choice	Last Choice
1	5	0	6
2	2	7	2
3	4	4	3

Candidate	Borda Count
1	
2	
3	

Voting Paradox

Candidate	First Choice	Second Choice	Last Choice
1	5	0	6
2	2	7	2
3	4	4	3

Candidate	Borda Count
1	10
2	11
3	12

Voting Paradox

Candidate	First Choice	Second Choice	Last Choice
1	5	0	6
2	2	7	2
3	4	4	3

Candidate	Borda Count
1	10
2	11
3	**12**

By the application of Representation Theory, we have learned that given any voting profile, any candidate can become the winner!

Thank you for listening!